Effect of Viscous Damping on Inelastic Design Spectra

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ABSTRACT:
Seismic regulations define design seismic ground motion as elastic spectra associated to 5-percent viscous damping. Reduction factors are applied to the 5-percent spectra to consider energy dissipation in inelastic structural systems. In some instances, however, viscous damping differs from 5 percent. For instance, ASCE Manual 113 requires that the structures of electrical substations be designed for 2-percent damping. Correction factors are prescribed for other than 5-percent damped systems. These factors are based on studies considering elastic behavior. However, the effects of viscous damping on inelastic spectra could differ significantly from those on elastic systems. This paper presents a study over the impact of viscous damping on spectra of inelastic structures with various types of hysteretic behavior. The seismic input consists of earthquake records from Californian and Mexico City sites. We propose simple equations for modifying inelastic spectra from 5 to other percents of critical damping, which depend on the ductility demand.

Keywords: spectra, inelastic, damping, hysteretic, degrading

1. INTRODUCTION
Most current codes stipulate elastic design spectra corresponding to 5% of critical damping. However, different damping ratios may be either prescribed or judged to be adequate for some structures or components. Thus, damping correction factors (DCFs) are applied to the 5%-damped. Early DCF equations were proposed by Arias and Husid (1962) and Newmark and Rosenblueth (1971). Recently, Cameron and Green (2007) have reviewed the literature on the topic and developed factors in terms of structural period, frequency content of the ground motion, and tectonic setting. All these studies deal with elastic spectra. However, current seismic design regulations recognize that, under severe events, the structures will undergo significant nonlinear deformations. On account of this beneficial inelastic behavior, results of elastic analyses are reduced by a factor, R. Typically, R is specified in terms of the type of system adopted to resist lateral forces and on the natural period of the structure. As an example, ASCE Manual 113 (2008) and IEEE Standard 693 (2006) for electrical substations, have adopted specific damping correction factors (DCF) independent of inelastic behavior. At the same time, Manual 113 stipulates R-factors ranging from 1.3 to 3. When the viscous damping differs from 5%, the DCFs specified for elastic spectra are applied to the R-reduced spectra. This study shows that this practice could be inaccurate and that DCFs also depend significantly on the types of inelastic behavior.

2. EFFECT OF VISCOUS DAMPING ON ELASTIC SYSTEMS
DCFs are developed by comparing elastic spectra for various damping ratios from ensembles of earthquake records. In this study, to evaluate the current IEEE DCFs (adopted by ASCE 113) and to develop factors applicable to nonlinear structures, we use two sets of earthquake records: one consisting in 87 records from Californian stiff to medium stiff sites, with dominant periods between
0.2s and 0.3s, and the second comprising 66 record from the soft lakebed of Mexico City with dominant periods of nearly 2s. The records of each set were separately normalized to have the same Arias Intensity and to yield average peak ground acceleration equal to that of gravity. Average 5%-damped elastic spectra of the two ensembles are presented in Figs. 1 and 2.

![Graph showing elastic calculated and approximate (IEEE approach) spectra, Californian Records](image1)

**Figure 1.** Elastic Calculated and Approximate (IEEE approach) Spectra, Californian Records

![Graph showing elastic calculated and approximate (IEEE approach) spectra, Mexico City Records](image2)

**Figure 2.** Elastic Calculated and Approximate (IEEE approach) Spectra, Mexico City Records

Fig. 1 compares the calculated average elastic spectra for the Californian records used in this study with approximate values resulting from using of the above IEEE CDFs for 2 and 10%-damped spectra. The excellent agreement between calculated and approximate spectra indicates that the IEEE CDFs are adequate for Californian stiff to medium stiff sites. Fig. 2 displays a similar comparison for the ensemble of Mexican records. The agreement is still very good except for periods in the vicinity of the site dominant period (2s): the IEEE factors underestimate for 2%-damping spectral peak and overestimate the peak for 10% damping. The reason is that records from the soft lakebed of Mexico City exhibit a much narrower frequency content than Californian records.
Standard IEEE 693 stipulates the following damping correction factors:

\[
DCF = \beta , \quad \text{for } 0 \leq f \leq 8 \text{ Hz} \quad (2.1)
\]

\[
DCF = 1 + 0.04(\beta - 1)(33 - f) , \quad \text{for } 8 \leq f \leq 33 \text{ Hz} \quad (2.2)
\]

\[
DCF = 1, \quad \text{for } f > 33 \text{ Hz} \quad (2.3)
\]

\[
\beta = 1.5173 - 0.3213 \ln(d) \quad (2.4)
\]

\(d = \text{percent of damping ratio}\)
\(f = \text{frequency in Hz = inverse of period, T, in seconds.}\)

3. EFFECT OF VISCOUS DAMPING ON INELASTIC NONDEGRADING SYSTEMS

In this study we consider the two types of inelastic force-displacement curves depicted in Fig. 3. The first curve represents nondegrading bilinear behavior with second slope equal to 2 percent of the initial one, yield strength \(F_y\), and initial stiffness \(k\), and with stable hysteretic cycles. The second curve shows total stiffness degradation associated to structures where steel X-bracing is the primary component to resist lateral forces (Bazán and Rosenblueth, 1974).

Our first finding is that appreciable better agreement between 5%-damped calculated spectra and the IEEE approximation for Mexican records is achieved when the spectrum yielding a ductility of unity is considered instead of the entirely elastic spectra, as illustrated in Fig. 4. To define a ductility-one spectrum we conducted nonlinear analysis of non-degrading systems to determine the yield resistance that results on an average ductility of 1.0. The maximum shear force induced by some records is smaller than the yield resistance producing a ductility smaller than unity, while other records generate a shear force larger than the yield resistance with an ensuing ductility bigger than 1.0. The difference between the two concepts is apparent when comparing the 4.4 calculated peak of the 5%-damped of the strictly elastic spectrum in Fig. 2 with the 5.3 calculated peak of the 5% ductility-one spectrum. Clearly, even a limited amount of inelastic behavior smoothes the peaks of narrow banded spectra. We conducted a similar exercise with the Californian records finding no appreciable difference between the 5%-damped strictly elastic spectrum and the ductility-one spectrum. Thus, we conclude that the IEEE approach is adequate for ductility-one spectra in both Californian and Mexico City sites.

**Figure 3.** Bilinear Non-Degraded (a) and Degrading Hysteretic Curves
To quantify the impact of viscous damping ratios on inelastic nondegrading structures, we have calculated inelastic spectra for average ductility demands of 2 and 4, using the ensembles of Californian and Mexico City records. Fig. 4 presents the calculated inelastic spectra of Californian records for an average ductility of 2, and viscous damping ratios of 2, 5 and 10%. Also presented in Fig. 4 are approximate spectra for 2 and 10% damping obtained by multiplying the 5%-damped spectrum by CDFs defined as follows:

\[
 DCF(\mu) = DCF(1) \left( \frac{1}{n(\mu)} \right) 
\]

(3.5)

\[
 n(\mu) = 1 + 1.2 \ln(\mu) 
\]

(3.6)
DCF(1) is the factor given by Eqs. (2.1) to (2.4) and includes the correction for damping ratio other than 5%.

Fig. 5 shows that Eqs. (3.5) and (3.6) lead to accurate estimates of the calculated inelastic spectra. Additional spectral comparisons are presented in Fig. 6 for $\mu = 4$ and Californian records, Fig. 7 for $\mu = 2$ and Mexican records, and Fig. 8 and for $\mu = 2$ and Mexican records. In all cases, the approximations provided by Eqs. (3.5) and (3.6) agree very well with the calculated inelastic spectra over the entire ranges of studied periods.

![Figure 6](image)

**Figure 6.** Inelastic Spectra for $\mu = 4$, Non-Degrading Systems, Californian Records

![Figure 7](image)

**Figure 7.** Inelastic Spectra for $\mu = 2$; Non-degrading Systems; Mexico City Records
4. EFFECT OF VISCOS DAMPING ON INELASTIC DEGRADING SYSTEMS

The hysteretic loops of most inelastic structures degrade appreciably under repeated load cycles. To assess the impact of viscous damping on degrading systems, we have considered the total stiffness behavior represented in Figure 3b. Degrading in this type of system is rather severe since for cycles of the same amplitude, no energy is dissipated by hysteresis after the first cycle. Using this degrading load displacement curve, we have repeated the calculations described in the previous section for calculated inelastic spectra for average ductility demands of 2 and 4, again using the ensembles of Californian and Mexico City records.

Fig. 9 shows the calculated and approximated inelastic spectra for an average ductility demand of 4. Examining this figure and the results for ductility demand of 2 (omitted for brevity), we find that CDF factors defined by Eq. (3.5) applies to approximate the spectra for 2 and 10% damping if Eq. (3.6) is replaced by:
\[ n(\mu) = 1 + 0.3 \ln(\mu) \] (4.7)

Similar results corresponding to Mexico City records are presented in Fig. 10. Examination of this figure and the results for ductility demand of 2 (omitted for brevity), we find that CDF factors are practically insensitive to the ductility demand. The approximation shown in Fig. 10 was obtained by straightforward use of the IEEE CDFs (Eqs. (2.1) to (2.4)). This observation, as well as the low coefficient of 0.3 affecting \(\ln(\mu)\) term in Eq. (4.7), reflect the limited hysteretic energy dissipation capability of systems with severe degradation. Thus, viscous damping plays a more significant role in the seismic response than in non-degrading systems.

5. CONCLUSION REMARKS

Step-by-step analyses of nonlinear single-degree-of-freedom systems reveal that the effects of viscous damping on bilinear systems depend on their ability to dissipate hysteretic energy. Thus, the modification of a design 5%-damped spectrum to represent a different damping ratio depends on the type of hysteretic behavior and on ductility demand.

The procedure of Standard IEEE 693 is very accurate for elastic spectra of Californian records. However, errors up to 19% occur for elastic spectra of events recorded in the soft lakebed of Mexico City. The IEEE approach becomes significantly more accurate when applied to the spectrum yielding a ductility of unity instead of the entirely elastic spectra. In our opinion, code design spectra to be reduced due to inelastic behavior are more correctly interpreted as being ductility-one.

Our study of non-degrading bilinear systems indicates that viscous damping correction factors (DCFs) developed to adjust elastic spectra must be appreciably modified in terms of the ductility demand. Eqs. (3.5) and (3.6) can be used for this purpose. As an example, Fig. 11 shows how inaccurate are the IEEE equations when directly applied to spectra for ductility 4 of Mexico City. In particular, spectral values can be underestimated by more than 20% for damping ratios of 10%.
Our analyses of systems with total stiffness degrading behavior indicates that changes in viscous damping have a similar effect as in entirely elastic systems. This result was expected since in the absence of significant hysteretic loops, viscous damping becomes the dominant source of energy dissipation.

REFERENCES